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**A GENERALIZED NETWORK FORMULATION FOR APERTURE
PROBLEMS**

Roger F. Harrington, et al

Syracuse University

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A GENERALIZED NETWORK FORMULATION FOR
APERTURE PROBLEMS

by

Roger F. Harrington
Joseph R. Maas

Department of
Electrical and Computer Engineering
Syracuse University
Syracuse, New York 13210

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13. ABSTRACT A general formulation for aperture problems is given in terms of the method of moments. It applies to any two regions isolated except for coupling through the aperture. The aperture characteristics are expressed in terms of two aperture admittance matrices, one for each region. The admittance matrix for one region is independent of the other region, and hence can be used for any problem involving that region and aperture. The solution can be represented by two generalized n-port networks connected in parallel with current sources. The current sources are related to the tangential magnetic field which exists over the aperture region when the aperture is closed by an electric conductor. Explicit formulations are given for two problems, that of an aperture in a conducting plane with plane-wave excitation, and that of a waveguide feeding an aperture in a conducting plane.			

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Aperture admittance						
Apertures						
Method of moments						
Plane conductors with apertures						
Transmission through apertures						
Waveguide-fed apertures						

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I. INTRODUCTION

The general problem of coupling through apertures has many specific applications, such as apertures in a conducting screen, waveguide-fed apertures, cavity-fed apertures, waveguide-to-waveguide coupling, waveguide-to-cavity coupling and cavity-to-cavity coupling. The literature on these problems is extensive. Many books, of which [1] to [3] are typical, discuss the problem and give references to some of the literature.

This report formulates the problem in terms of a moment solution of the operator equation. An application of the equivalence principle separates the problem into two parts, namely, the regions on each side of the aperture. The only coupling is through the aperture, whose characteristics can be expressed by aperture admittance matrices, one for each region. These admittance matrices depend only on the region being considered, being independent of the other region. The aperture coupling is then expressible as the sum of the two independent aperture admittance matrices, with source terms related to the incident magnetic field. This result can be interpreted in terms of generalized networks as two n -port networks connected in parallel with current sources. The resultant solution is equivalent to an n -term variational solution.

Since the problem is divided into two mutually exclusive parts, one can separately solve a few canonical problems, such as apertures in conducting screens, in waveguides, and in cavities, and then combine them in the various permutations mentioned above. Computer programs can be developed for treating broad classes of canonical problems, such as apertures of arbitrary shape in conducting planes, in square waveguides, and in rectangular cavities. Such programs can then serve as broad and versatile

-
- [1] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Company, New York, 1960.
 - [2] R. F. Harrington, "Time-Harmonic Electromagnetic Fields," McGraw-Hill Book Company, New York, 1961.
 - [3] N. Amitay, V. Galindo, C. P. Wu, "Theory and Analysis of Phased Array Antennas," Wiley-Interscience, New York, 1972.

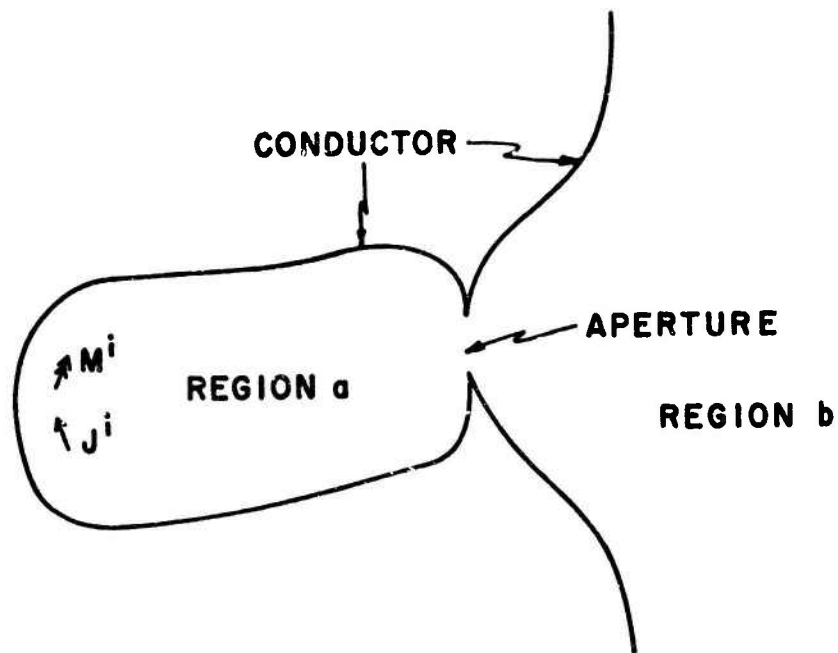


Fig. 1. The general problem of two regions coupled by an aperture.

tools for designing electromagnetic networks with aperture coupling.

II. GENERAL FORMULATION

Figure 1 represents the general problem of aperture coupling between two regions, called region a and region b. In region a there are impressed sources \underline{J}^i , \underline{M}^i , and region b is assumed source free. The more general case of sources in both region a and region b can be treated as the superposition of two problems, one with sources in region a only, plus one with sources in region b only. Each region of Fig. 1 is shown to be bounded by an electric conductor, although other types of electromagnetic isolation may be used. Region a is shown closed and region b is shown open to infinity, although each region may be open or closed. The equivalence principle [2, Sec. 3-5] is used to divide the problem into two equivalent problems, as shown in Fig. 2. In region a, the field is produced by the sources \underline{J}^i , \underline{M}^i , plus the equivalent magnetic current

$$\underline{M} = \underline{n} \times \underline{E} \quad (1)$$

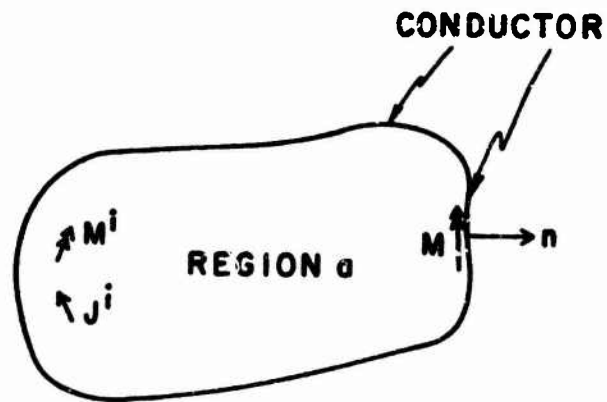
over the aperture region, with the aperture covered by an electric conductor. In region b, the field is produced by the equivalent magnetic current $-\underline{M}$ over the aperture region, with the aperture covered by an electric conductor. The fact that the equivalent current in region a is $+\underline{M}$ and that in region b is $-\underline{M}$ ensures that the tangential component of electric field is continuous across the aperture. The remaining boundary condition to be applied is continuity of the tangential component of magnetic field across the aperture.

The tangential component of magnetic field in region a over the aperture, denoted \underline{H}_t^a , is the sum of that due to the impressed sources, denoted \underline{H}_t^i , plus that due to the equivalent sources \underline{M} , denoted $\underline{H}_t^a(\underline{M})$, that is

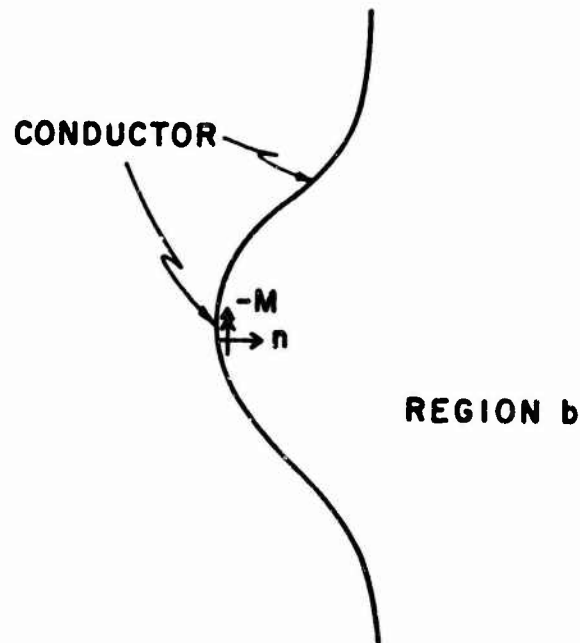
$$\underline{H}_t^a = \underline{H}_t^i + \underline{H}_t^a(\underline{M}) \quad (2)$$

Note that \underline{H}_t^i and $\underline{H}_t^a(\underline{M})$ are both computed with a conductor covering the aperture. A similar equation holds for region b, except that the equivalent sources $-\underline{M}$ are the only sources. Hence, the tangential component of magnetic field in region b over the aperture is

$$\underline{H}_t^b = \underline{H}_t^b(-\underline{M}) = -\underline{H}_t^b(\underline{M}) \quad (3)$$



(a) EQUIVALENCE FOR REGION a.



(b) EQUIVALENCE FOR REGION b.

Fig. 2. The original problem divided into two equivalent problems.

where $\underline{H}_t^b(\underline{M})$ is computed with a conductor covering the aperture. The last equality in (3) is a consequence of the linearity of the \underline{H}_t^b operator. The true solution is obtained when \underline{H}_t^a of (2) equals \underline{H}_t^b of (3), or

$$\underline{H}_t^a(\underline{M}) + \underline{H}_t^b(\underline{M}) = -\underline{H}_t^i \quad (4)$$

This is the basic operator equation for determining the equivalent magnetic current \underline{M} .

If (4) were satisfied exactly, we would have the true solution. We use the method of moments [4] to obtain an approximate solution. Define a set of expansion functions $\{\underline{M}_n, n=1,2,\dots,N\}$, and let

$$\underline{M} = \sum_n V_n \underline{M}_n \quad (5)$$

where the coefficients V_n are to be determined. Substitute (5) into (4) and use the linearity of the \underline{H}_t operators to obtain

$$\sum_n V_n \underline{H}_t^a(\underline{M}_n) + \sum_n V_n \underline{H}_t^b(\underline{M}_n) = -\underline{H}_t^i \quad (6)$$

Next, define a symmetric product

$$\langle \underline{A}, \underline{B} \rangle = \iint_{\text{apert.}} \underline{A} \cdot \underline{B} \, ds \quad (7)$$

and a set of testing functions $\{\underline{W}_m, m=1,2,\dots,N\}$, which may or may not be equal to the expansion functions. We take the symmetric product of (6) with each testing function \underline{W}_m , and use the linearity of the symmetric product to obtain the set of equations

$$\sum_n V_n \langle \underline{W}_m, \underline{H}_t^a(\underline{M}_n) \rangle + \sum_n V_n \langle \underline{W}_m, \underline{H}_t^b(\underline{M}_n) \rangle = -\langle \underline{W}_m, \underline{H}_t^i \rangle \quad (8)$$

$m=1,2,\dots,N$. Solution of this set of linear equations determines the coefficients V_n and the magnetic current \underline{M} according to (5). Once \underline{M} is known, the fields and field-related parameters may be computed by standard methods.

[4] R. F. Harrington, "Field Computation by Moment Methods," Macmillan Company, New York, 1968.

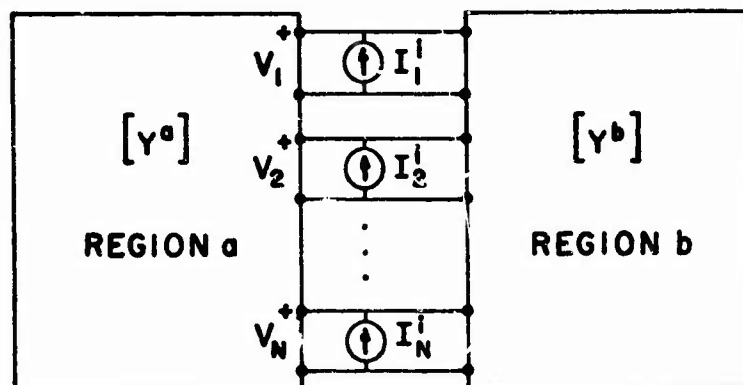


Fig. 3. The generalized network interpretation of equation (13).

The above solution can be put into matrix notation as follows:
Define an admittance matrix for region a as

$$[Y^a] = [\langle -W_m, H_t^a(M_n) \rangle]_{N \times N} \quad (9)$$

and an admittance matrix for region b as

$$[Y^b] = [\langle -W_m, H_t^b(M_n) \rangle]_{N \times N} \quad (10)$$

The minus signs are placed in (9) and (10) on the basis of power considerations. Define a source vector

$$\vec{I}^1 = [\langle W_m, H_t^1 \rangle]_{N \times 1} \quad (11)$$

and a coefficient vector

$$\vec{V} = [V_n]_{N \times 1} \quad (12)$$

Now the matrix equation equivalent to equations (8) is

$$[Y^a + Y^b] \vec{V} = \vec{I}^1 \quad (13)$$

This can be interpreted in terms of generalized networks as two networks $[Y^a]$ and $[Y^b]$ in parallel with the current source \vec{I}^1 , as shown in Fig. 3. The resultant voltage vector

$$\vec{V} = [Y^a + Y^b]^{-1} \vec{I}^1 \quad (14)$$

is then the vector of coefficients which give \underline{M} according to (5).

It is important to note that computation of $[Y^a]$ involves only region a, and computation of $[Y^b]$ involves only region b. Hence, we have divided the problem into two parts, each of which may be formulated independently. Once $[Y]$ is computed for one region, it may be combined with $[Y]$ for any other region, making it useful for a wide range of problems. For example, the same aperture admittance matrix for radiation into half-space would be useful for plane-wave excitation of the aperture, waveguide excitation, and cavity excitation.

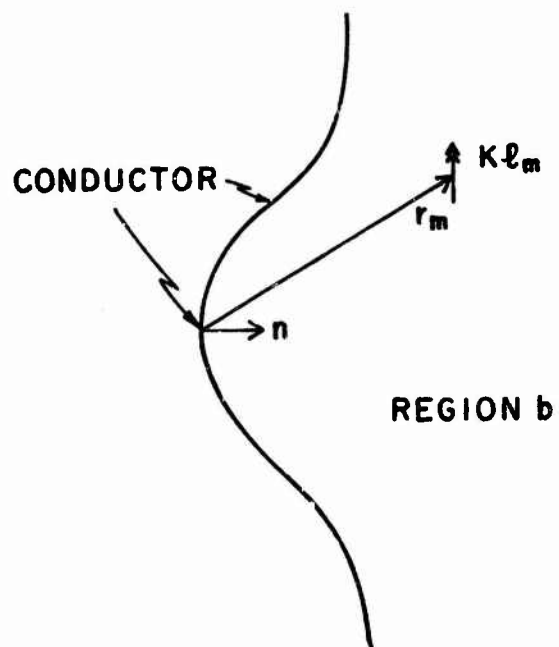


Fig. 4. The adjoint problem for determining H_m at r_m .

III. LINEAR MEASUREMENT

A linear measurement is defined as a number which depends linearly on the source. Examples of linear measurements are components of the field at a point, voltage along a given contour, and current crossing a given surface. Measurements made in region b will depend linearly only on the equivalent current $\underline{\underline{M}}$. Measurements made in region a will depend linearly on the impressed sources $\underline{\underline{J}}^i$, $\underline{\underline{M}}^i$, as well as on the equivalent current $\underline{\underline{M}}$. We now illustrate these concepts with a particular example.

Consider the measurement (computation) of a component H_m of magnetic field at a point $\underline{\underline{r}}_m$ in region b. It is known that this component can be obtained by placing a magnetic dipole $K\ell_m$ at $\underline{\underline{r}}_m$, and applying the reciprocity theorem to its field and to the original field [2, Sec. 3-8]. The original field in region b is given by the solution to Fig. 2b. The problem involving the magnetic dipole, called the adjoint problem, is shown in Fig. 4. Application of the reciprocity theorem to these two cases yields

$$H_m K\ell_m = - \iint_{\text{apert.}} \underline{\underline{M}} \cdot \underline{\underline{H}}^m ds \quad (15)$$

Here $\underline{\underline{H}}^m$ is the magnetic field from $K\ell_m$ in the presence of a complete conductor, and H_m is the component in the direction of $K\ell_m$ of the magnetic field at $\underline{\underline{r}}_m$ due to $\underline{\underline{M}}$ in the presence of a complete conductor. To evaluate (15), substitute for $\underline{\underline{M}}$ from (5) and obtain

$$H_m K\ell_m = \sum_n V_n \langle -\underline{\underline{M}}_n, \underline{\underline{H}}^m \rangle \quad (16)$$

This can be written in matrix form as

$$H_m K\ell_m = \tilde{I}^m V \quad (17)$$

where \tilde{I}^m is the transpose of a measurement vector

$$\tilde{I}^m = [\langle -\underline{\underline{M}}_n, \underline{\underline{H}}^m \rangle]_{N \times 1} \quad (18)$$

Note that the elements of \tilde{I}^m are similar in form to those of \tilde{I}^1 given by (11), except that $\underline{\underline{M}}_n$ replaces $\underline{\underline{W}}_n$. The minus sign difference reflects

the fact that the equivalent source in region b is $-\underline{\underline{M}}$, in contrast to that in region a which is $+\underline{\underline{M}}$. Now substitute (14) into (17) to obtain

$$\underline{\underline{H}}_m \underline{\underline{K}}_m = \underline{\underline{I}}^m [\underline{\underline{Y}}^a + \underline{\underline{Y}}^b]^{-1} \underline{\underline{I}}^1 \quad (19)$$

If the magnetic dipole is of unit moment, then (19) gives $\underline{\underline{H}}_m$ at $\underline{\underline{r}}_m$ directly.

Every linear measurement in region b will be of the form (19). For example, if a component of $\underline{\underline{E}}$ at $\underline{\underline{r}}_m$ were desired, we would place an electric dipole at $\underline{\underline{r}}_m$ and apply reciprocity. In general, a linear measurement involves applying reciprocity to the original problem and to an adjoint problem. A determination of the sources of the adjoint problem is a part of the formulation of the problem.

If a linear measurement is made in region a, it will involve a contribution from the impressed sources $\underline{\underline{J}}^1$, $\underline{\underline{M}}^1$ added to that from the equivalent sources $\underline{\underline{M}}$. For example, instead of (19) we would have

$$\underline{\underline{H}}_m \underline{\underline{K}}_m = \underline{\underline{H}}_m^1 \underline{\underline{K}}_m + \underline{\underline{I}}^m [\underline{\underline{Y}}^a + \underline{\underline{Y}}^b]^{-1} \underline{\underline{I}}^1 \quad (20)$$

where $\underline{\underline{H}}_m^1$ is the magnetic field from $\underline{\underline{J}}^1$, $\underline{\underline{M}}^1$ in the presence of a complete conductor. Also, in region a we would define the measurement vector to be

$$\underline{\underline{I}}^m = [\langle \underline{\underline{M}}_n, \underline{\underline{H}}^m \rangle]_{N \times 1} \quad (21)$$

instead of (18), because the equivalent sources are $+\underline{\underline{M}}$ in region a in contrast to $-\underline{\underline{M}}$ in region b. Note that it is the difference field $\underline{\underline{H}} - \underline{\underline{H}}^1$ in region a (due to $\underline{\underline{M}}$) that is directly analogous to the transmitted field $\underline{\underline{H}}$ in region b (due to $-\underline{\underline{M}}$).

IV. TRANSMITTED POWER

A quadratic measurement is one which depends quadratically on the sources. Examples of quadratic measurements are components of the Poynting vector at a point, power crossing a given surface, and energy within a given region. A particular quadratic measurement of considerable interest

is the power transmitted through the aperture, which we now consider.

The complex power P_t transmitted through the aperture is basically

$$P_t = \iint_{\text{apert.}} \underline{E} \times \underline{H}^* \cdot \underline{n} \, ds \quad (22)$$

where the asterisk denotes complex conjugate. Substituting from (1), we have

$$P_t = \iint_{\text{apert.}} \underline{M} \cdot \underline{H}^* \, ds \quad (23)$$

This involves only the tangential component of \underline{H} , which in region b we denote by $\underline{H}_t^b(-\underline{M})$. For \underline{M} we use the linear combination (5) and obtain

$$\underline{H}_t^b(-\underline{M}) = - \sum_n V_n \underline{H}_t^b(\underline{M}_n) \quad (24)$$

Substituting this for \underline{H} and (5) for \underline{M} into (23), we obtain

$$P_t = - \sum_m \sum_n V_m V_n^* \iint_{\text{apert.}} \underline{M}_m \cdot \underline{H}_t^{b*}(\underline{M}_n) \, ds \quad (25)$$

If \underline{M}_m are real, the conjugate operations can be taken outside the integrals. Moreover, if $\underline{M}_m = \underline{W}_m$ (Galerkin's method), then the negative of the integrals in (25) are Y_{mn}^{b*} as defined by (10), and

$$P_t = \sum_m \sum_n V_m V_n^* Y_{mn}^{b*} \quad (26)$$

This can be written in matrix form as

$$P_t = \tilde{V} [Y^b]^* \tilde{V}^* \quad (27)$$

Note that this is the usual formula for power into network $[Y^b]$ of Fig. 3.

V. APERTURES IN PLANE CONDUCTORS

Consider a conducting plane covering the $z=0$ plane except for an aperture, as shown in Fig. 5. The two regions $z>0$ and $z<0$ are identical half spaces, and hence their admittance matrices are the same. Therefore, we let

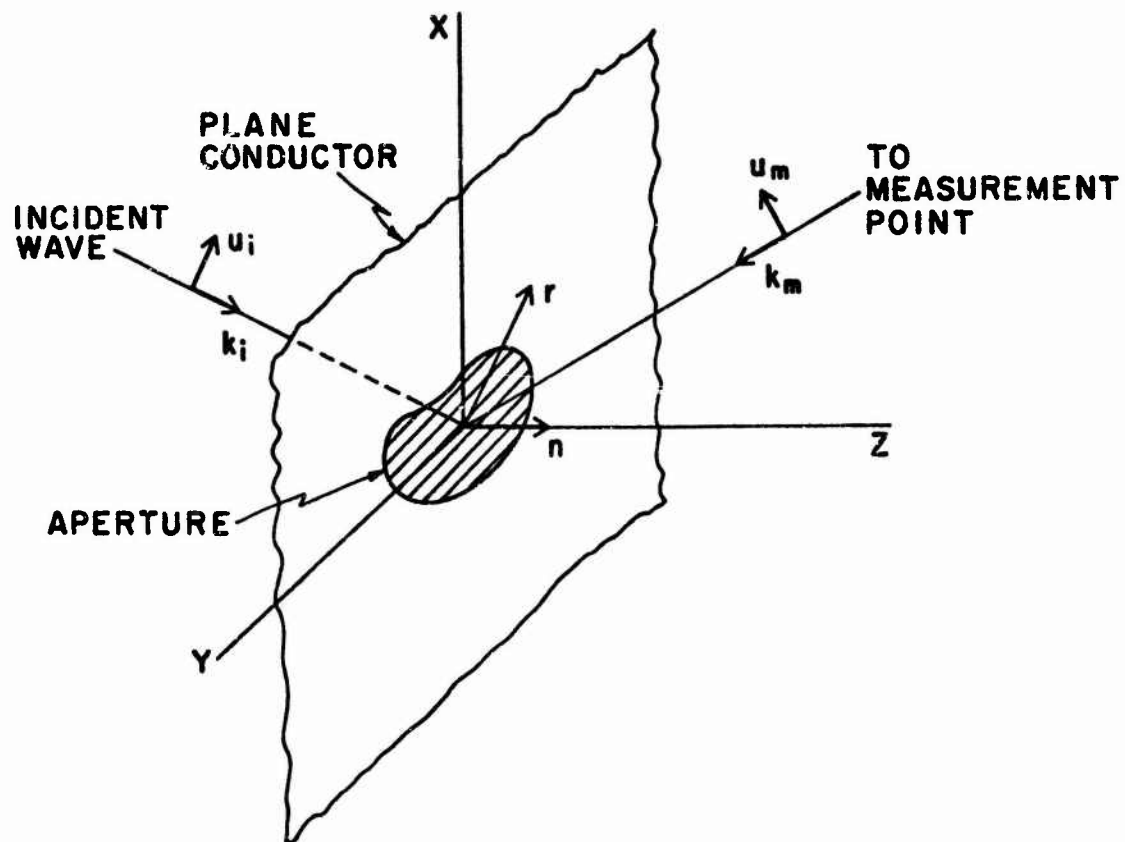


Fig. 5. Aperture in a plane conductor.

$$[Y^a + Y^b] = 2[Y^{hs}] \quad (28)$$

where $[Y^{hs}]$ denotes the aperture admittance for the aperture opening into half space, say $z > 0$. When the aperture is covered by a conductor, the $z=0$ plane is a complete conducting plane, and image theory applies. The magnetic current expansion functions are on the surface of the $z=0$ plane. Their images are equal to them and are also on the $z=0$ plane [2, Sec. 3-6]. The result is that $[Y^{hs}]$ is the admittance matrix obtained using expansion functions $2\tilde{M}_n$ radiating into free space everywhere. This problem is dual to that for the impedance matrix of a plane conductor, a problem considered recently in the literature [5].

The original excitation of the aperture is by the impressed sources \tilde{J}^i, \tilde{M}^i in the region $z < 0$. The impressed field \tilde{H}_t^i used in the operator equation (4) is the tangential magnetic field due to \tilde{J}^i, \tilde{M}^i with the aperture covered by a conductor (Fig. 2a). In this case the $z=0$ plane is a complete conductor, and image theory again applies. The result is that the tangential component of \tilde{H} over the $z=0$ plane when it is covered by a conductor is just twice what it is for the same sources in free space. Hence,

$$\tilde{H}_t^i = 2\tilde{H}_t^{i0} \quad (29)$$

where \tilde{H}_t^{i0} is the tangential component of the magnetic field over the aperture due to the sources \tilde{J}^i, \tilde{M}^i in free space. The components of the excitation vector \tilde{I}^i defined by (11) are now

$$I_m^i = 2 \iint_{\text{apert.}} \tilde{W}_m \cdot \tilde{H}_t^{i0} ds \quad (30)$$

where \tilde{W}_m is the m th testing function.

A case of special interest is that of plane wave excitation. A unit plane wave is given by

-
- [5] N. N. Wang, J. H. Richmond, M. C. Gilreath, "Sinusoidal Reaction Formulation for Radiation and Scattering from Conducting Surfaces," IEEE Trans. on Antennas and Propagation, vol. AP-23, No. 3, pp. 376-382, May 1975.

$$\underline{H}^{10} = \underline{u}_1 e^{-jk_1 \cdot \underline{r}} \quad (31)$$

where \underline{u}_1 is a unit vector specifying the direction of \underline{H}^{10} , k_1 is the propagation vector of magnitude $2\pi/\lambda$ and pointing in the direction of propagation, and \underline{r} is the radius vector to an arbitrary field point. These vectors are shown in Fig. 5. The components (30) of the plane-wave excitation vector are then

$$\underline{P}_m^1 = 2 \iint_{\text{apert.}} \underline{W}_m \cdot \underline{u}_1 e^{-jk_1 \cdot \underline{r}} ds \quad (32)$$

The symbol \underline{P}^1 has been used for this particular vector to distinguish it from the more general excitation vector (30).

Similar simplifications apply to the adjoint (measurement) problem. For the evaluation of a component of magnetic field at a point \underline{r}_m , a magnetic dipole $K\underline{l}_m$ is placed at the measurement point \underline{r}_m . This radiates in the presence of a complete conductor over the $z=0$ plane, and hence, analogous to (29), we have

$$\underline{H}_t^m = 2\underline{H}_t^{m0} \quad (33)$$

Here \underline{H}_t^m denotes the tangential component of \underline{H} over the aperture from $K\underline{l}_m$ when the $z=0$ plane is covered by a conductor, and \underline{H}_t^{m0} denotes that from $K\underline{l}_m$ when it radiates into free space. The components of the measurement vector \underline{I}^m defined by (18) are now

$$\underline{I}_n^m = -2 \iint_{\text{apert.}} \underline{M}_n \cdot \underline{H}_t^{m0} ds \quad (34)$$

where \underline{M}_n is the n th expansion function.

A case of special interest is that of far-field measurement. This is obtained by a procedure dual to that used for radiation and scattering from conducting wires [6]. To obtain a component of \underline{H} on the radiation sphere, we take a source $K\underline{l}_m$ perpendicular to \underline{r}_m and let $\underline{r}_m \rightarrow \infty$.

[6] R. F. Harrington, "Matrix Methods for Field Problems," Proc. IEEE, vol. 55 No. 2, pp. 136-149, February 1967.

At the same time we adjust $K\ell_m$ so that it produces a unit plane wave in the vicinity of the origin. The required dipole moment is given by

$$\frac{1}{K\ell_m} = \frac{-j\omega\epsilon}{4\pi r_m} e^{-jk r_m} \quad (35)$$

and the plane-wave field it produces in the vicinity of the origin is

$$\underline{H}_m^{mo} = \underline{u}_m e^{-jk_m \cdot \underline{r}} \quad (36)$$

Here \underline{u}_m is a unit vector in the direction of \underline{H}_m^{mo} , \underline{k}_m is the propagation vector, and \underline{r} is the radius vector to an arbitrary field point. Again these vectors are shown in Fig. 5. The components (34) of the far-field measurement vector are then

$$\underline{p}_n^m = -2 \iint_{\text{apert.}} \underline{M}_n \cdot \underline{u}_m e^{-jk_m \cdot \underline{r}} d\mathbf{s} \quad (37)$$

The symbol \underline{p}^m is used for this particular measurement vector to distinguish it from the more general measurement vector (34). The far-zone magnetic field is now given by (19) with $K\ell_m$ given by (35), $\underline{I}^m = \underline{p}^m$, $\underline{I}^1 = \underline{p}^1$, and the aperture admittance given by (28). Hence

$$\underline{H}_m = \frac{-j\omega\epsilon}{8\pi r_m} e^{-jk r_m} \underline{p}^m [Y^{hs}]^{-1} \underline{\hat{p}}^1 \quad (38)$$

The usual two radiation components H_θ and H_ϕ are obtained by orienting $\underline{K\ell}_m$ in the θ and ϕ directions, respectively.

A parameter sometimes used to express the transmission characteristics of an aperture is the transmission cross section τ . It is defined as that area for which the incident wave contains sufficient power to produce the radiation field \underline{H}_m by omnidirectional radiation over half space. For unit incident magnetic field, this is

$$\tau = 2\pi r_m^2 |\underline{H}_m|^2 \quad (39)$$

Substituting from (38), we obtain

$$\tau = \frac{\omega^2 \epsilon^2}{32\pi} |\mathbf{P}^m[\mathbf{Y}^{hs}]^{-1} \mathbf{P}^1|^2 \quad (40)$$

Note that τ depends upon the polarization and direction of the incident wave (via \mathbf{P}^1), and upon the polarization measured and direction to the measurement point (via \mathbf{P}^m).

Another parameter used to express the transmission characteristics of an aperture is the transmission coefficient T , defined as

$$T = \frac{P_{\text{trans.}}}{P_{\text{inc.}}} \quad (41)$$

where $P_{\text{trans.}}$ is the time-average power transmitted by the aperture, and $P_{\text{inc.}}$ is the free space power incident on the aperture. The incident power is

$$P_{\text{inc.}} = \eta S \cos \theta_{\text{inc.}} \quad (42)$$

where $\eta = \sqrt{\mu/\epsilon}$ is the intrinsic impedance of free space, S is the aperture area, and $\theta_{\text{inc.}}$ is the angle between \mathbf{k}_i and \mathbf{n} . The transmitted power is

$$P_{\text{trans.}} = \text{Re}(P_t) \quad (43)$$

where $\text{Re}(P_t)$ denotes the real part of P_t , given by (27) with $[\mathbf{Y}^b] = [\mathbf{Y}^{hs}]$. Hence

$$T = \frac{1}{\eta S \cos \theta_{\text{inc.}}} \text{Re}(\mathbf{V}[\mathbf{Y}^{hs}]^* \mathbf{V}) \quad (44)$$

Note that T depends on both the direction of incidence and on the polarization of the incident wave.

Finally, because of symmetry about the $z=0$ plane, the difference field $\underline{\mathbf{H}} - \underline{\mathbf{H}}^1$ which exists in the region $z < 0$ is simply related to the transmitted field which exists in the region $z > 0$. The difference field

in the region $z < 0$ is produced by an equivalent current \underline{M} on a plane conductor over the $z=0$ plane. By image theory, it is also the field produced in the region $z < 0$ by the source $2\underline{M}$ in free space. As noted earlier, the transmitted field in the region $z > 0$ is produced by the source $-\underline{2M}$ in free space. Hence, the difference field in the region $z < 0$ and the negative of the transmitted field in the region $z > 0$ are both produced by the same magnetic current $2\underline{M}$ radiating in free space. In other words, the difference field is the negative of the analytic continuation of the transmitted field into the region $z < 0$. This applies to electric fields as well as to magnetic fields.

VI. WAVEGUIDE-FED APERTURES

Consider now a uniform waveguide feeding an aperture in a conducting plane, as shown in Fig. 6. In general, the aperture may be of different size and shape than the waveguide cross section. The half-space region $z > 0$ is the same as in the previous problem, Fig. 5, and the analysis of the preceding section applies. An analysis of the waveguide region is given below.

Let the excitation of the waveguide be a source which produces a single mode, of unit amplitude, incident on the aperture. This mode (usually the dominant mode) is denoted by the index 0. The field tangential to the z -direction can then be expressed in modal form as [2, Sec. 8-1]

$$\begin{aligned} \underline{E}_t &= e^{-\gamma_0 z} \underline{e}_0 + \sum_1 \Gamma_1 e^{\gamma_1 z} \underline{e}_1 \\ \underline{H}_t &= \gamma_0 e^{-\gamma_0 z} \underline{u}_z \times \underline{e}_0 - \sum_1 \Gamma_1 \gamma_1 e^{\gamma_1 z} \underline{u}_z \times \underline{e}_1 \end{aligned} \quad (45)$$

It is assumed that all modes, TE and TM, are included in the summation. The γ_1 are modal propagation constants

$$\gamma_1 = \begin{cases} j\beta_1 = jk \sqrt{1 - (\lambda/\lambda_1)^2} & \lambda < \lambda_1 \\ \alpha_1 = k_1 \sqrt{1 - (\lambda_1/\lambda)^2} & \lambda > \lambda_1 \end{cases} \quad (46)$$

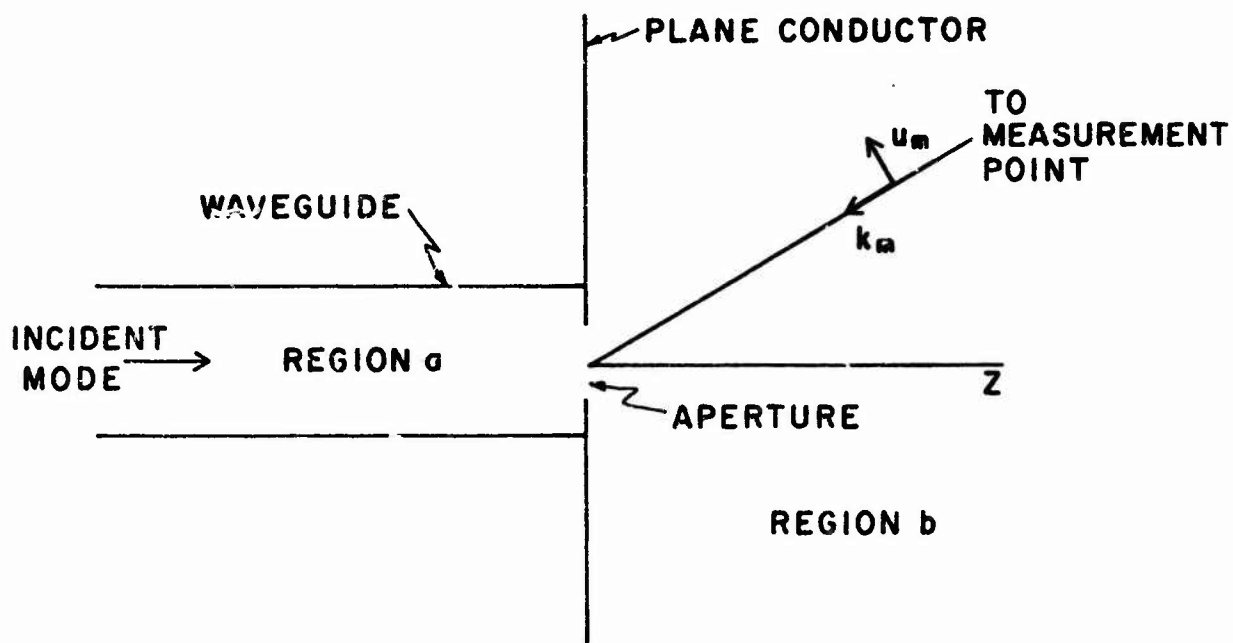


Fig. 6. Waveguide-fed aperture in a conducting plane.

where λ_i is the i -th mode cut-off wavelength, and $k_i = 2\pi/\lambda_i$ is the i -th mode cut-off wavenumber. The Y_i are the modal characteristic admittances

$$Y_i = \begin{cases} Y_i/j\omega\mu & \text{TE modes} \\ j\omega\epsilon/Y_i & \text{TM modes} \end{cases} \quad (47)$$

Γ_0 is the reflection coefficient for the 0-th mode, and Γ_i is the complex amplitude of the $-z$ traveling component of the i -th mode. The \vec{e}_i are normalized modal vectors, so that the modal orthogonality relationships are

$$\iint_{\text{guide}} \vec{e}_i \cdot \vec{e}_j ds = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases} \quad (48)$$

where the integration is over the waveguide cross section.

To evaluate the aperture admittance (9) in the waveguide region, we consider a single expansion function M_n on the $z=0$ plane in the waveguide region. The tangential field produced by M_n will be of the form (45), except that there is no incident wave. Hence, this field is

$$\begin{aligned} \vec{E}_t^a(M_n) &= \sum_i A_{ni} e^{\gamma_i z} \vec{e}_i \\ \vec{H}_t^a(M_n) &= - \sum_i A_{ni} Y_i e^{\gamma_i z} \vec{u}_z \times \vec{e}_i \end{aligned} \quad (49)$$

where the A_{ni} are modal amplitudes. At $z=0$ we have

$$\vec{M}_n = \vec{u}_z \times \vec{E}_t^a \Big|_{z=0} = \sum_i A_{ni} \vec{u}_z \times \vec{e}_i \quad (50)$$

Multiply each side of this equation scalarly by $\vec{u}_z \times \vec{e}_j$ and integrate over the waveguide cross section, obtaining

$$\iint_{\text{guide}} \vec{M}_n \cdot \vec{u}_z \times \vec{e}_j ds = \sum_i A_{ni} \iint_{\text{guide}} (\vec{u}_z \times \vec{e}_i) \cdot (\vec{u}_z \times \vec{e}_j) ds \quad (51)$$

By orthogonality (48), all terms of the summation are zero except the $i=j$ term. Hence,

$$A_{ni} = \iint_{\text{apert.}} \underline{M}_{ni} \cdot \underline{u}_z \times \underline{e}_1 ds \quad (52)$$

We have replaced the integral over the waveguide cross section by one over the aperture, since \underline{M}_{ni} exists only in the aperture region. The elements of the aperture admittance matrix (9) are now given by

$$Y_{mn}^{wg} = - \iint_{\text{apert.}} \underline{W}_m \cdot \underline{H}_t^a(\underline{M}_{ni}) ds \quad (53)$$

where the superscript wg denotes waveguide. The \underline{H}_t^a of (53) is given by the second equation of (49) evaluated at $z=0$, or

$$Y_{mn}^{wg} = \sum_i A_{ni} Y_i \iint_{\text{apert.}} \underline{W}_m \cdot \underline{u}_z \times \underline{e}_1 ds \quad (54)$$

Now define the constants

$$B_{mi} = \iint_{\text{apert.}} \underline{W}_m \cdot \underline{u}_z \times \underline{e}_1 ds \quad (55)$$

which are similar in form to the A_{ni} of (52). The elements (54) then are given by

$$Y_{mn}^{wg} = \sum_i A_{ni} B_{mi} Y_i \quad (56)$$

Hence, all elements of the waveguide admittance matrix $[Y^{wg}]$ are linear combinations of the modal characteristic admittances Y_i . For Galerkin's method, $\underline{W}_m = \underline{M}_m$ and the A_{ni} and B_{mi} are equal.

We next evaluate the equivalent magnetic current \underline{M} , given by (5). The incident field is given by the first term on the right-hand side of (45). When the aperture is covered by a conductor, the waveguide is terminated by a conducting plane. According to image theory, the tangential magnetic field at $z=0$ is then just twice the incident wave, or

$$\underline{H}_t^1 = 2Y_0 \underline{u}_z \times \underline{e}_0 \quad (57)$$

This is the \underline{H}_t^1 used in (11) to evaluate the excitation vector \vec{I}^1 . Hence, the components of the excitation vector are

$$I_m^1 = 2Y_0 \iint_{\text{apert.}} \underline{W}_m \cdot \underline{u}_z \times \underline{e}_0 ds = 2Y_0 B_{m0} \quad (58)$$

The total aperture admittance matrix is

$$[Y^a + Y^b] = [Y^{wg} + Y^{hs}] \quad (59)$$

where $[Y^{wg}]$ is the waveguide aperture admittance and $[Y^{hs}]$ is the half-space aperture admittance. The coefficient matrix \vec{V} is given by (14) with the admittance matrix given by (59), or

$$\vec{V} = [Y^{wg} + Y^{hs}]^{-1} \vec{I}^1 \quad (60)$$

Finally, the equivalent magnetic current \underline{M} is given by (5) where the coefficients V_n are the components of \vec{V} .

Once \underline{M} is found, the modal amplitudes Γ_1 in (45) can be evaluated from (1) and the orthogonality properties of the modes. From (1) and (45), we have

$$\underline{M} = \underline{u}_z \times \underline{E}_t \Big|_{z=0} = \underline{u}_z \times \underline{e}_0 + \sum_1 \Gamma_1 \underline{u}_z \times \underline{e}_1 \quad (61)$$

Now multiply each side scalarly by $\underline{u}_z \times \underline{e}_j$ and integrate over the waveguide cross section. By the orthogonality relationships (48), all terms of the summation vanish except the term $i=j$. The result is

$$\iint_{\text{guide}} \underline{M} \cdot \underline{u}_z \times \underline{e}_1 ds = \begin{cases} 1 + \Gamma_0 & i = 0 \\ \Gamma_1 & i \neq 0 \end{cases} \quad (62)$$

Here the integration over the guide can be changed to that over the aperture because $\underline{M} = 0$ except in the aperture. Substituting for \underline{M} from (5) into (62), and using the definitions (52), we have

$$\sum_n V_n A_{no} = 1 + \Gamma_o \quad (63)$$

$$\sum_n V_n A_{ni} = \Gamma_i \quad i \neq 0$$

Finally, by defining modal measurement vectors as

$$\vec{A}_i = [A_{ni}]_{N \times 1} \quad (64)$$

and using (60), we can write (63) as

$$1 + \Gamma_o = \tilde{A}_o [Y^{wg} + Y^{hs}]^{-1} \vec{I}^1 \quad (65)$$

and, for $i \neq 0$,

$$\Gamma_i = \tilde{A}_i [Y^{wg} + Y^{hs}]^{-1} \vec{I}^1 \quad (66)$$

The parameter of most interest is Γ_o , the reflection coefficient of the incident mode. This is often expressed in terms of an admittance

$$Y_{ap} = \frac{1 - \Gamma_o}{1 + \Gamma_o} Y_o \quad (67)$$

which is the equivalent aperture admittance seen by the incident mode.

The region $z > 0$ for waveguide-fed apertures is the same half-space region as existed in the previous problem of an aperture in a conducting plane. Hence, evaluation of the fields in terms of \vec{M} in this region is done the same way as in Section V. For example, the \vec{u}_m component of the far-zone magnetic field at a point \vec{r}_m is given by

$$\vec{H}_m = \frac{-j\omega\epsilon}{4\pi r_m} e^{-jk r_m} \tilde{P}^m [Y^{wg} + Y^{hs}]^{-1} \vec{I}^1 \quad (68)$$

which is (38) with the term $[2Y^{hs}]^{-1} \vec{P}^1$ replaced by (60). The excitation vector \vec{I}^1 has elements given by (58), and the far-field measurement vector \tilde{P}^m has elements given by (37). The power gain pattern is the ratio of the radiation intensity in a given direction to the radiation intensity

which would exist if the total power P_t were radiated uniformly over half space, or

$$G = \frac{2\pi r_m^2 \eta |H_m|^2}{\text{Re}(P_t)} \quad (69)$$

Substituting for H_m from (68), we have

$$G = \frac{\omega^2 \epsilon^2 \eta}{8\pi \text{Re}(P_t)} |\tilde{P}^m [Y^{wg} + Y^{hs}]^{-1} \tilde{I}|^2 \quad (70)$$

where P_t is given by (27). Note that this gain is a function of the H component measured, as well as direction to the field point.

VII. DISCUSSION

A general formulation for aperture problems using the method of moments has been given. The solution is expressed in terms of aperture admittance matrices, one for each region. While the exposition has been written in terms of a single aperture the formulation applies equally well to multiple apertures. The extension of the theory to several regions is also relatively straightforward. An example of this more general case is many waveguides radiating into half space, a problem treated by Galindo and Wu [3]. Another example is a coaxial line feeding a cavity, which in turn radiates into half space through an aperture. In this case the generalized network equivalent would be a network representing the cavity, connected through some ports to a network representing the coaxial line, and connected through other ports to a network representing half space.

Explicit formulations have been given for two problems, that of an aperture in a conducting plane with plane-wave excitation, and that of a waveguide feeding an aperture in a conducting plane. General computer programs have been developed for rectangular apertures in conducting planes, and for rectangular apertures in rectangular waveguides. These programs will be described in future reports. It is also planned to apply the general theory to develop computer programs for rectangular

apertures in rectangular cavities. We will then have programs which can be used for computing the behavior of rectangular apertures coupling two regions which are any permutation of these three cases. Other applications of the general theory, such as to apertures and cavities of arbitrary shape, and to multiple aperture problems, are contemplated for future work.

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